PH 712 Probability and Statistical Inference

Part III: Mutiple Random Variables

Zhiyang Zhou (zhou67@uwm.edu, zhiyanggeezhou.github.io)

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Characterizing the joint distribution of multiple random variables

- The joint cdf of $X_1, \dots, X_n$: $F_{X_1,\dots,X_n}(x_1, \dots, x_n) = \Pr(X_1 \le x_1, \dots, X_n \le x_n)$, $(x_1, \dots, x_n) \in \mathbb{R}^n$
- Discrete $X_1, \dots, X_n$
  - Joint pmf $p_{X_1,\dots,X_n}(x_1, \dots, x_n) = \Pr(X_1 = x_1, \dots, X_n = x_n)$, $(x_1, \dots, x_n) \in \mathbb{R}^n$
  - $\mathrm{supp}(X_1, \dots, X_n) = \{(x_1, \dots, x_p) \in \mathbb{R}^n : p_{X_1,\dots,X_n}(x_1, \dots, x_n) > 0\}$
  - Marginal pmf of X_i:

$$p_{X_i}(x_i) = \sum_{x_1,\dots,x_{i-1},x_{i+1},\dots,x_n} p_{X_1,\dots,X_n}(x_1, \dots, x_n)$$

- Continuous $X_1, \dots, X_n$
  - Joint pdf $f_{X_1,\dots,X_n}(x_1, \dots, x_n) = (\partial^n/\partial x_1 \cdots \partial x_n) F_{X_1,\dots,X_n}(x_1, \dots, x_n)$, $(x_1, \dots, x_n) \in \mathbb{R}^n$
  - $\mathrm{supp}(X_1, \dots, X_n) = \{(x_1, \dots, x_n) \in \mathbb{R}^n : f_{X_1,\dots,X_n}(x_1, \dots, x_n) > 0\}$
  - Marginal pdf of X_1:

$$f_{X_i}(x_i) = \int_{\mathbb{R}^{n-1}} f_{\boldsymbol{X}}(\boldsymbol{x}) \mathrm{d}x_1 \cdots \mathrm{d}x_{i-1} \mathrm{d}x_{i+1} \cdots \mathrm{d}x_n$$

(Mutual) independence

- RVs $X_1, \dots, X_n$ are (mutually) independent $\Leftrightarrow$

$$F_{X_1,\dots,X_n}(x_1, \dots, x_n) = \prod_{i=1}^n F_{X_i}(x_i)$$

  - Joint pmf $p_{X_1,\dots,X_n}(x_1, \dots, x_n) = \prod_{i=1}^n p_{X_i}(x_i)$ (for discrete $X_1, \dots, X_n$)
  - Joint pdf $f_{X_1,\dots,X_n}(x_1, \dots, x_n) = \prod_{i=1}^n f_{X_i}(x_i)$ (for continuous $X_1, \dots, X_n$)
- (Optional) The mgf of X_i exists for all i, say $M_{X_i}(t_i)$. Then $X_1, \dots, X_n$ are (mutually) independent $\Rightarrow$ $M_{X_1,\dots,X_n}(t_1, \dots, t_n) = \prod_{i=1}^n M_{X_i}(t_i)$